Calculate $15^{2021} \pmod{17}$. (Hint: You may want to choose a different representation of 15 in mod 17.)

Solution: Instead of using brute repeated exponentiation, we can convert this to a more manageable form: $(-2)^{2021} \pmod{17}$ since $15 \equiv -2 \pmod{17}$. Now we notice that $(-2)^4 \equiv 16 \equiv -1 \pmod{17}$. Hence,

$$\begin{aligned}
15^{2021} &\equiv (-2)^{2021} && \pmod{17} \\
&\equiv ((-2)^4)^{505} \cdot -2 && \pmod{17} \\
&\equiv (-1)^{505} \cdot -2 && \pmod{17} \\
&\equiv -1 \cdot -2 && \pmod{17} \\
&\equiv 2 && \pmod{17}
\end{aligned}$$